Semi-analytical solutions for dispersive shock waves in colloidal media

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Abstract:

The diffractive resolution of a discontinuity at the edge of an optical beam in a colloidal suspension of spherical dielectric nanoparticles by a collisionless shock, or dispersive shock wave, is studied. The interaction of the nanoparticles is modelled as a hard-sphere gas with the Carnahan-Starling formula used for the gas compressibility. The governing equation is a focusing nonlinear Schrödinger-type equation with an implicit nonlinearity. It is found that the discontinuity is resolved through the formation of a dispersive shock wave which forms before the eventual onset of modulational instability. A semi-analytical solution is developed in the $(1+1)$ dimensional case by approximating the dispersive shock wave as a train of uniform solitary waves. A semi-analytical solution is also developed for a $(2+1)$ dimensional circular dispersive shock wave for the case in which the radius of the bore is large. Depending on the value of the background packing fraction, three qualitatively different solitary wave amplitude versus jump height diagrams are possible. For large background packing fractions a single stable solution branch occurs. At moderate values an S-shaped response curve results, with multiple solution branches, while for small values the upper solution branch separates from the middle unstable branch. Hence, for low to moderate values of the background packing fraction the dispersive shock bifurcates from the low to the high power branch as the jump height, the height of the input beam's edge discontinuity, is increased. These multiple steady-state response diagrams, also typically found in combustion applications, are unusual in applications involving solitary waves. The predictions of the semi-analytical theory are found to be in excellent agreement with numerical solutions of the governing equations for both line and circular dispersive shock waves. The method used represents a new technique for obtaining semi-analytical results for a dispersive shock wave in a focusing medium.